Cp/Cpk vs. Pp/Ppk

What is the difference?
Which one should I use?

Richard Engel

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Background

The original definition and interpretation of Capability was from *Statistical Quality Control Handbook Capability* by the Western Electric Company (1956). In the book, capability is defined as "the natural or undisturbed performance after extraneous influences are eliminated. This is determined by plotting data on a control chart." This means that engineers used process capability studies to eliminate all special causes from a process before using that process on the "shop floor" and proved it using Rbar/d2 and control charts. The index Cpk is a more recent index that also accounts for the centering of the process, and is predominately utilized today. (1)

Terms

AIAG is considered by many as the "standard" definition of the various indices AIAG identifies the following Indices:

- Cp = Capability Index
- Pp = Performance Index
- Cpk = Capability Index which accounts for process centering
- Ppk = Performance Index which accounts for process centering (2)

Formulas

$$Cp = \frac{(USL - LSL)}{6 * \hat{\sigma}_{\bar{R}/d_2}}$$

$$Pp = \frac{(USL - LSL)}{6 * \hat{\sigma}_{S}}$$

$$Cpl = \frac{(Mean - LSL)}{3 * \hat{\sigma}_{\bar{R}/d_2}}$$

$$Ppl = \frac{(Mean - LSL)}{3 * \hat{\sigma}_{S}}$$

$$Cpu = \frac{(USL - Mean)}{3 * \hat{\sigma}_{\bar{R}/d_2}}$$

$$Ppu = \frac{(USL - Mean)}{3 * \hat{\sigma}_{S}}$$

$$Cpk = Min(Cpl, Cpu)$$

$$Ppk = Min(Ppl, Ppu)$$

Note the only difference is the estimator of Sigma

Formula Interpretation

$\hat{\sigma}$ Sigma Hat = Estimator of Sigma

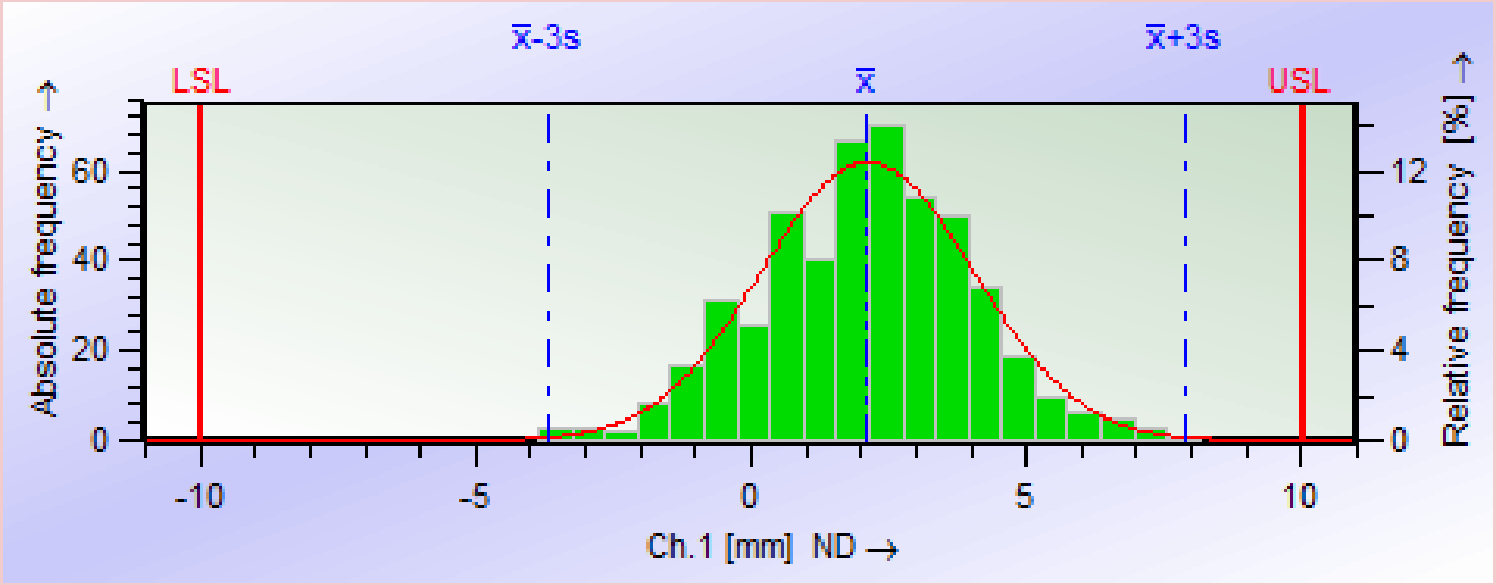
$\hat{\sigma}_{\overline{R}/d_2}$ Sigma Hat R bar/ d2 It is from the range chart and is the average range divided by a constant from statistical tables. Alternatively sometimes s bar/c4 from the s chart is used.

$\hat{\sigma}_s$ Sigma Hat S = Sigma from all the samples using the formula $s = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \overline{x})^2}{n-1}} = \hat{\sigma}_s$

C Indices use the Subgroup Data

P Indices use ALL the data to calculate Sigma

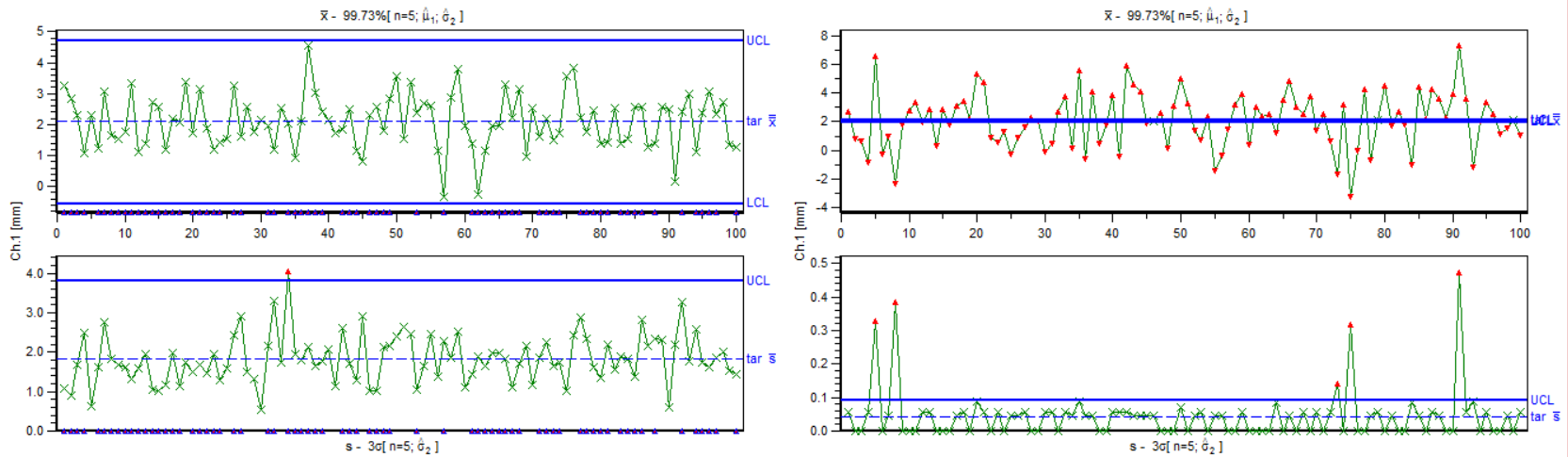
Comparison Normal Stable Data



	C Index	P Index
P	1.71	1.73
Pk	1.36	1.36

There is insignificant Difference between the C and the P indices in this case

Effect of Sampling



The same data is presented in Random sub-groups on the left and "artificially sorted" sub-groups on the right; This could represent sub-groups taken from a process with tool to tool variation or tool wear where the variation in any sub-group is insignificant compared to the overall variation in the process

Unsorted			Sorted	
C Index	P Index		C Index	P Index
1.71	1.73	P	79.93	1.73
1.36	1.36	Pk	63.18	1.36

As the within subgroup variation is reduced the overall C indices are greatly exaggerated

Calculations Cp

	Left Sample	Right Sample
$\bar{R}$	4.542	0.097
d_2	2.326	2.326
$\bar{R}/d_2$	1.953	0.042
(USL - LSL)	20.0	20.0
$6\hat{\sigma}_{\bar{R}/d_2}$	11.718	0.250
$Cp = \frac{(USL - LSL)}{6 * \hat{\sigma}_{\bar{R}/d_2}}$	1.71	79.93

Low R Bar leads to high Cp

Calculations Pp

	Left Sample	Right Sample
$\hat{\sigma}_s$	1.932	1.932
(USL - LSL)	20.0	20.0
$6\hat{\sigma}$	11.592	11.592
$Pp = \frac{(USL - LSL)}{6 * \sigma_s}$	1.73	1.73

Equal $\hat{\sigma}_s$ Means equal Pp's

Evolving Definitions

AIAG 1992 SPC Manual

The capability Index (e.g. Cpk) is additionally useful for determining whether or not a process is capable of meeting customer requirements (the original intent of the capability index) It should be pointed out that this additional use should not be applied to performance indices.... (3)

AIAG 2006 PPAP Manual

Initial Process Studies... When historical data are available or enough initial data exist to plot a control chart ... Cpk can be calculated when the process is stable. Otherwise, for processes with known and predictable special causes and output meeting specifications Ppk should be used. (4)

Ppk moved from Should Not in 1992 to Should in 2006

Evolving Definitions

ISO 21747:2006

This relatively recent spec defines Statistical methods for process performance and capability statistics. Basically it defines an identical formula for Pp and Cp and likewise identical formulas for Cpk and Ppk. The estimators for 6 sigma are replaced by a reference interval. This reference interval is the interval bounded by the 99.865%quartile and the 0.135% quartile. For the Normal distribution this interval = 6S. This specification also allows to alternatively use the within subgroup estimators$(\bar{R}/d_2)$ for the reference interval calculation. I think this further confuses an issue which otherwise is clarified by this standard. The user will be safe in using the standard calculations proposed for the reference interval calculations. The standard also notes that " a quantitative comparison of the performance and capability indices according to the different methods is not feasible." (5)

Does it Matter C vs.. P?

I have checked with several associates looking for examples where they got in trouble using C indices vs. P indices and I have tried to create the contrived distribution to show that an acceptable C index was really a BAD distribution. Every example that I have reviewed that met this test failed one of the two basic rules of the C index – The distribution **must be normal and stable.** For the P indices, even when not stable, accurate results can be achieved using sigma if the distribution is normal and for non-normal distributions accurate results can be achieved with the reference interval calculation. This is also known as the Percentile method in advanced Statistical packages such as those offered by Q-DAS (my personal favorite). The C and P indices are identical when stable and normal. The C may be grossly inaccurate otherwise. Why take a chance – use the P indices, preferably with the reference interval calculation.

Summary

- C Indices were invented in the 1950's when calculation of sigma was difficult and time consuming.
- C and P indices are equivalent when Process is stable and Normal.
- C Indices underestimate total process variation when the within subgroup variation is low.
- **Generally the P indices are the best indices to use.**
- Where appropriate software is available, use the reference interval to estimate the total variation.
- It is important for each company to have written standards defining the appropriate use of Capability indices and the formulas associated with each.

Caution

Most modern SPC packages allow any of a number of formulas to be associated with each index. It is the user's responsibility to be aware of this and define the proper formula to each index. These should be per a company standard defining same and agreed to with your customer.

References

(1) Santy, Woody. *November 2008*. Tech. no. ASQ Section 1206. ASQ, 05 Dec. 2008. Web. 19 Apr. 2010. <http://www.asqsection1206.org/1/post/2008/12/iso-90012008.html>.

(2) AIAG. "Definition of Process Measures." *Statistical Process Control*. AIAG, 1995. pg 80.. 2nd Printing.

(3) AIAG. "description of Conditions and Assumptions." *Statistical Process Control*. AIAG, 1995. pg 81. 2nd Printing.

(4) AIAG. "Quality Indices." *PPAP* AIAG, 2006. pg 8.. *4th Edition* .

(5) ISO Reference Interval Pg 4 and General Geometric Method pg 24 ISO 21747:2006(E) 2006-07-01 First Edition